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Sheffer Operation in Ortholattices *

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Abstract

We introduce the concept of Sheffer operation in ortholattices and, more generally, in lattices with antitone involution. By using this, all the fundamental operations of an ortholattice or a lattice with antitone involution are term functions built up from the Sheffer operation. We list axioms characterizing the Sheffer operation in these lattices.

Key words: Ortholattice, orthocomplementation, lattice with antitone involution, Sheffer operation.

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The concept of Sheffer operation (the so-called Sheffer stroke in [1]) was introduced by H. M. Sheffer in 1913. H. M. Sheffer [3] showed that all Boolean functions could be obtained from a single binary operation as term operations. In what follows, we are going to show that this works also in ortholattices and, more generally, in lattices with antitone involution and we will set up an equational axiomatization of this Sheffer operation.

Our basic concepts are taken from [1] and [2]. By a *bounded lattice* we mean a lattice with least element $\mathbf{0}$ and greatest element $\mathbf{1}$. Let $\mathcal{L} = (L; \vee, \wedge)$ be a lattice. A mapping $x \mapsto x^\perp$ is called an antitone involution on $\mathcal{L}$ if

$x \leq y$ implies $y^\perp \leq x^\perp$ (antitone)

$x^{\perp\perp} = x$ (involution).

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