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Ideals, Congruences and Annihilators on Nearlattices*

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Abstract

By a nearlattice is meant a join-semilattice having the property that every principal filter is a lattice with respect to the semilattice order. We introduce the concept of (relative) annihilator of a nearlattice and characterize some properties like distributivity, modularity or 0-distributivity of nearlattices by means of certain properties of annihilators.

Key words: Nearlattice; semilattice; ideal; congruence; distributivity; modularity; 0-distributivity; annihilator.

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1 Introduction

Algebraic structures being join-semilattices with respect to the induced order relation appear frequently in algebraic logic. For example, implication algebras, introduced by J. C. Abbott [1], describe algebraic properties of the logical connective implication in the classical propositional logic. Implication algebras have a very nice structure: with respect to the induced order, they are join-semilattices, principal filters of which are Boolean algebras. Analogously, for various logics of quantum mechanics the corresponding algebraic structures have a semilattice structure with principal filters being special lattices.

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This fact motivated us to describe $\vee$-semilattices where every principal filter is a lattice. They are called nearlattices (see e.g. [3, 5, 6, 11, 14, 15, 16, 17]).

More precisely, we studied the following structures.

Definition 1 A semilattice $\mathcal{N} = (N; \vee)$, where for each $a \in N$ the principal filter $[a) = \{x \in N; a \leq x\}$ is a lattice with respect to the induced order $\leq$ of $\mathcal{N}$, is called a *nearlattice.*

It has been shown [4, 11] that nearlattices can be considered as algebras with one ternary operation. Moreover, nearlattices considered as algebras of type (3) form an equational class: indeed, if $x, y, z \in N$ for a nearlattice $\mathcal{N}$, the element $(x \vee z) \wedge (y \vee z)$ is correctly defined since both $x \vee z, y \vee z \in [z)$ and $[z)$ is a lattice, and the following holds:

Proposition 1 ([4]) *Let $\mathcal{N} = (N; \vee)$ be a nearlattice. Define a ternary operation by $m(x, y, z) = (x \vee z) \wedge (y \vee z)$ on N. Then $m(x, y, z)$ is an everywhere defined operation and the following identities are satisfied:*

(P1) $m(x, y, x) = x$;

(P2) $m(x, x, y) = m(y, y, x)$;

(P3) $m(m(x, x, y), m(x, x, y), z) = m(x, x, m(y, y, z))$;

(P4) $m(x, y, p) = m(y, x, p)$;

(P5) $m(m(x, y, p), z, p) = m(x, m(y, z, p), p)$;

(P6) $m(x, m(y, y, x), p) = m(x, x, p)$;

(P7) $m(m(x, x, p), m(x, x, p), m(y, x, p)) = m(x, x, p)$;

(P8) $m(m(x, x, z), m(y, y, z), z) = m(x, y, z)$.

Conversely, let $\mathcal{N} = (N; m)$ be an algebra of type (3) *satisfying* (P1)–(P7)*. If we define $x \vee y = m(x, x, y)$, then $(N; \vee)$ is a join-semilattice and for each $p \in N$, $([p); \leq)$ is a lattice, where for $x, y \in [p)$ their infimum is $x \wedge y = m(x, y, p)$. Hence $(N; \vee)$ is a nearlattice. If, moreover, $\mathcal{N} = (N; m)$ satisfies also* (P8)*, then the correspondence between nearlattices and algebras $(N; m)$ satisfying* (P1)–(P8) *is one-to-one.*

Thus nearlattices similarly as lattices have two faces and we shall alternate in our investigations between them depending which one will be more convenient.

The following notions of distributivity for nearlattices have been introduced in [4]:

Definition 2 Let $\mathcal{N} = (N; m)$ be an algebra of type (3). We call $\mathcal{N}$ *distributive* if it satisfies the identity

(D1) $m(x, m(y, y, z), p) = m(m(x, y, p), m(x, y, p), m(x, z, p))$.

If $\mathcal{N}$ satisfies the identity

(D2) $m(x, x, m(y, z, p)) = m(m(x, x, y), m(x, x, z), p)$,

it is called *dually distributive.*

It is expected that both notions are related in the case of nearlattices. Indeed, one can prove the following statement:

Proposition 2 ([4]) *Let* $\mathcal{N} = (N; m)$ *be an algebra of type* (3) *satisfying* (P1)–(P7). *Then the following conditions are equivalent:*

(1) $\mathcal{N}$ *is distributive;*

(2) $\mathcal{N}$ *is dually distributive;*

(3) *in the associated semilattice, every principal filter is a distributive lattice.*

Due to the previous description of distributivity for nearlattices, we are able to get very simple arguments to prove that in a distributive nearlattice $\mathcal{N}$, every ideal of $\mathcal{N}$ is a congruence class.

2 Ideals and congruence classes on distributive nearlattices

The concept of an ideal in a distributive nearlattice was defined in [10]:

Definition 3 A subset $\emptyset \neq I \subseteq N$ of a nearlattice $\mathcal{N} = (N; m)$ is called an *ideal* if

(I1) $m(x, x, y) \in I$ for all $x, y \in I$;

(I2) $m(x, y, p) \in I$ for all $x \in I$ and $y, p \in N$ with $p \leq x$.

Note that I is an ideal of $\mathcal{N}$ if and only if it is a downset closed under suprema with respect to the induced order of $\mathcal{N}$.

Lemma 1 *A subset* $\emptyset \neq I \subseteq N$ *of a nearlattice* $\mathcal{N} = (N; \vee)$ *is an ideal if and only if it satisfies the following two conditions*

(i1) $x, y \in I \Rightarrow x \vee y \in I$;

(i2) $x \in I,\ a \leq x \Rightarrow a \in I$.

Proof It is clear. $\square$

Example 1 Let $\mathcal{N} = (\{x, x \vee y, y, p, q, 1\}; \vee)$ be a nearlattice whose diagram is depicted in Fig. 1. The set $I = \{x, x \vee y, y\}$ is clearly an ideal on $\mathcal{N}$.

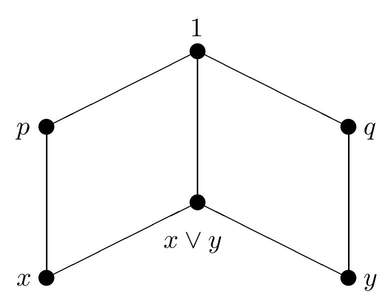

Fig. 1

By a *congruence* on a nearlattice $\mathcal{N} = (N; m)$ we mean an equivalence relation Θ on N such that for all $x_1, x_2, y_1, y_2, z_1, z_2 \in N$ we have that $\langle x_1, x_2\rangle \in \Theta$, $\langle y_1, y_2\rangle \in \Theta$, $\langle z_1, z_2\rangle \in \Theta$ imply

$$\langle m(x_1, y_1, z_1), m(x_2, y_2, z_2)\rangle \in \Theta.$$

This concept can be translated for the alternative description of a nearlattice as follows:

Lemma 2 *Let $\mathcal{N} = (N; \vee)$ be a nearlattice. Then Θ is a congruence on $\mathcal{N}$ if and only if it is an equivalence relation on N which satisfies the following implication $(*)$:*
$\langle x_1, x_2\rangle, \langle y_1, y_2\rangle \in \Theta \;\Rightarrow\; \langle x_1 \vee y_1, x_2 \vee y_2\rangle \in \Theta$, and $\langle x_1 \wedge y_1, x_2 \wedge y_2\rangle \in \Theta$, whenever $x_1 \wedge y_1, x_2 \wedge y_2$ are defined.

Proof $(\Rightarrow)$: Let Θ be a congruence on $\mathcal{N}$. Let $\langle x_1, x_2\rangle \in \Theta$ and $\langle y_1, y_2\rangle \in \Theta$. Then, by definition of congruence on $\mathcal{N}$, $\langle m(x_1, x_1, y_1), m(x_2, x_2, y_2)\rangle \in \Theta$, i.e. $\langle x_1 \vee y_1, x_2 \vee y_2\rangle \in \Theta$. Now, we observe the following property of Θ :

(P) If $x \leq y$, $\langle x, y\rangle \in \Theta$ and $x \wedge z$ exists, then $\langle x \wedge z, y \wedge z\rangle \in \Theta$.

Indeed, we have $\langle m(x, z, x \wedge z), m(y, z, x \wedge z)\rangle \in \Theta$, where

$$m(x, z, x \wedge z) = (x \vee (x \wedge z)) \wedge (z \vee (x \wedge z)) = x \wedge z$$

and

$$m(y, z, x \wedge z) = (y \vee (x \wedge z)) \wedge (z \vee (x \wedge z)) = y \wedge z,$$

and hence $\langle x \wedge z, y \wedge z\rangle \in \Theta$.

Now, assume that $\langle x_1, x_2\rangle, \langle y_1, y_2\rangle \in \Theta$, and $x_1 \wedge y_1, x_2 \wedge y_2$ exists. Then $\langle x_1, x_1 \vee x_2\rangle \in \Theta$ and since $x_1 \wedge y_1$ exists, we have $\langle x_1 \wedge y_1, (x_1 \vee x_2) \wedge y_1\rangle \in \Theta$ by (P). Analogously, $\langle y_1, y_1 \vee y_2\rangle \in \Theta$ entails

$$\langle (x_1 \vee x_2) \wedge y_1, (x_1 \vee x_2) \wedge (y_1 \vee y_2)\rangle \in \Theta.$$

Therefore

$$\langle x_1 \wedge y_1, (x_1 \vee x_2) \wedge (y_1 \vee y_2)\rangle \in \Theta.$$

Similarly we can show that $\langle x_2 \wedge y_2, (x_1 \vee x_2) \wedge (y_1 \vee y_2)\rangle \in \Theta$. Consequently, due to transitivity of Θ we obtain $\langle x_1 \wedge y_1, x_2 \wedge y_2\rangle \in \Theta$.
$(\Leftarrow)$: Let Θ be an equivalence relation on N satisfying $(*)$. Let $\langle x_1, x_2\rangle$, $\langle y_1, y_2\rangle$, $\langle z_1, z_2\rangle \in \Theta$. Then $\langle x_1 \vee z_1, x_2 \vee z_2\rangle$, $\langle y_1 \vee z_1, y_2 \vee z_2\rangle \in \Theta$, and hence also

$$\langle (x_1 \vee z_1) \wedge (y_1 \vee z_1), (x_2 \vee z_2) \wedge (y_2 \vee z_2)\rangle \in \Theta,$$

i.e. $\langle m(x_1, y_1, z_1), m(x_2, y_2, z_2)\rangle \in \Theta$, thus Θ is a congruence on $\mathcal{N}$. □

We can show that for distributive nearlattices, the ideals are related to congruences in the same way as it is for lattices (see e.g. [9]).

Theorem 1 *Let $\mathcal{N} = (N; \vee)$ be a distributive nearlattice. Then each ideal I of $\mathcal{N}$ is a congruence class of $\Theta_I \in Con\mathcal{N}$, defined by*

$$\langle x, y\rangle \in \Theta_I \quad \textit{iff} \quad \textit{there exists } c \in I \textit{ such that } x \vee c = y \vee c.$$

Proof Of course, Θ_I is reflexive and symmetric. Suppose $\langle a, b\rangle \in \Theta_I$ and $\langle b, c\rangle \in \Theta_I$. Then $a \vee x = b \vee x$ and $b \vee y = c \vee y$ for some $x, y \in I$. Since I is an ideal, we have $x \vee y \in I$. Thus $a \vee x \vee y = b \vee x \vee y = c \vee x \vee y$, whence $\langle a, c\rangle \in \Theta_I$, i.e. Θ_I is an equivalence on N.

Let $\langle a, b\rangle \in \Theta_I$ and $c \in N$. Then there exists $x \in I$ such that $a \vee x = b \vee x$ and thus $a \vee c \vee x = b \vee c \vee x$, hence $\langle a \vee c, b \vee c\rangle \in \Theta_I$. Using transitivity of Θ_I, we easily obtain that Θ_I is compatible with the operation $\vee$.

Now, let $\langle a, b\rangle \in \Theta_I$, $\langle c, d\rangle \in \Theta_I$ and let $a \wedge c$, $b \wedge d$ are defined. Then $a \vee x = b \vee x$ and $c \vee y = d \vee y$ for some $x, y \in I$. Applying distributivity of N, we have

$$(a \vee x) \wedge (c \vee y) = (a \wedge c) \vee (x \wedge c) \vee (a \wedge y) \vee (x \wedge y) = (a \wedge c) \vee z,$$

where $z = (x \wedge (c \vee y)) \vee (y \wedge (a \vee x)) \in I$. Analogously,

$$(b \vee x) \wedge (d \vee y) = (b \wedge d) \vee (x \wedge d) \vee (b \wedge y) \vee (x \wedge y) = (b \wedge d) \vee z,$$

which gives $\langle a \wedge c, b \wedge d\rangle \in \Theta_I$, i.e. Θ_I is compatible with a partial operation $\wedge$. Applying Lemma 2, we have shown that Θ_I is a congruence on $\mathcal{N}$.

Further, suppose $a, b \in I$. By (i1), $a \vee b \in I$ and since $a \vee (a \vee b) = b \vee (a \vee b)$, we have $\langle a, b\rangle \in \Theta_I$. Conversely, let $a \in I$ and $\langle a, c\rangle \in \Theta_I$. Then there exists $x \in I$ such that $a \vee x = c \vee x$. But $a \vee x \in I$, whence $c \vee x \in I$. Since $c \leq c \vee x$, by (i2) we have $c \in I$, which yields $I = [a]_{\Theta_I}$, i.e. I is a class of Θ_I. $\square$

Corollary 1 *Each ideal of a nearlattice $\mathcal{N} = (N; \vee)$ is a class of at least one congruence if and only if $\mathcal{N}$ is distributive.*

Proof If $\mathcal{N}$ is distributive and I is its ideal then, by Theorem 1, I is a class of the congruence Θ_I.

Conversely, let $\mathcal{N}$ be not distributive. Then, by Proposition 2, there exists a principal filter $[b)$ which is not a distributive lattice, i.e. it contains N_5 or M_3 (see Fig. 2).

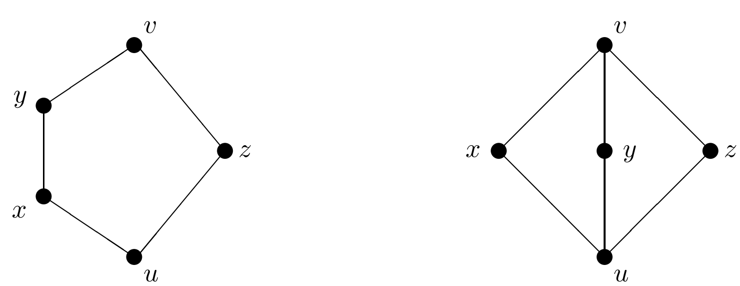

Fig. 2

In both cases, one can easily prove that $(x] = I(x) = \{a \in N; a \leq x\}$ is an ideal on nearlattice $\mathcal{N}$ which is not a class of any congruence Θ on $\mathcal{N}$. Indeed, let $(x]$ be a class of congruence Θ on $\mathcal{N}$. Since $u, x \in (x]$ we have $\langle u, x\rangle \in \Theta$ (see Fig. 2). So $\langle u \vee z, x \vee z\rangle \in \Theta$, i.e. $\langle z, v\rangle \in \Theta$. Further, $\langle z \wedge y, v \wedge y\rangle \in \Theta$ because $z \wedge y$ and $v \wedge y$ exists in $[u)$. Hence $\langle u, y\rangle \in \Theta$, which yields $y \in (x]$, a contradiction. □

3 Annihilators on nearlattices

The aim of this section is to show that annihilators can be used for a characterization of distributivity or modularity of nearlattices in the way similar to that for lattices, see e.g. [7, 12, 13]. However, the concept of a relative annihilator must be defined in a slightly different way from that for lattices [2, 8, 12].

Definition 4 Let $\mathcal{N} = (N; \vee)$ be a nearlattice and $a, b, x, z \in N$. By a *relative annihilator of a with respect to b* we mean the set $\langle a, b\rangle = \{z \in N; z \leq x$ where $a \wedge x$ exists and $a \wedge x \leq b\}$.

Remark 1 It means that our relative annihilator in a nearlattice is in fact a downset of a relative annihilator as defined in [7, 12, 13]. The reason is that e.g. for $\langle q, y\rangle$ of the nearlattice from Example 1 we have $(x \vee y) \wedge q \leq y$ thus $x \vee y \in \langle q, y\rangle$ but $x \leq x \vee y$ and $x \wedge q$ is not defined. Hence, we must extend the original concept into a downset.

Theorem 2 *Let $\mathcal{N} = (N; \vee)$ be a nearlattice. The following conditions are equivalent:*

(i) *$\mathcal{N}$ is distributive;*

(ii) *$\langle a, b\rangle$ is an ideal of $\mathcal{N}$ for all $a, b \in N$;*

(iii) *$\langle a, b\rangle$ is an ideal of $\mathcal{N}$ for each $b \leq a$.*

Proof (i)⇒(ii): Let $\mathcal{N}$ be distributive and $a, b \in N$. Suppose $z \in \langle a, b\rangle$ and $y \leq z$. Then obviously $y \in \langle a, b\rangle$. If $z, y \in \langle a, b\rangle$ then $z \leq x_1$ with $a \wedge x_1 \leq b$ and $y \leq x_2$ with $a \wedge x_2 \leq b$ (for some $x_1, x_2 \in N$). Thus $z \vee y \leq x_1 \vee x_2$. It is evident that all considered meets exist and due to distributivity of $\mathcal{N}$,

$$(x_1 \vee x_2) \wedge a = (x_1 \wedge a) \vee (x_2 \wedge a) \leq b.$$

Hence $x_1 \vee x_2 \in \langle a, b\rangle$ and thus also $z \vee y \in \langle a, b\rangle$, i.e. $\langle a, b\rangle$ is an ideal of $\mathcal{N}$.

(ii)⇒(iii) is trivial. Prove (iii)⇒(i). Let $a \in N$ and $x, y, z \in [a)$. Then $y \wedge x, z \wedge x$ exist and $(y \wedge x) \vee (z \wedge x) \leq x$. Hence, by (iii), $\langle x, (y \wedge x) \vee (z \wedge x)\rangle$ is an ideal I of $\mathcal{N}$. Since $x \wedge y \leq (y \wedge x) \vee (z \wedge x)$, we have $y \in I$. Analogously, $x \wedge z \leq (y \wedge x) \vee (z \wedge x)$, thus $z \in I$ and hence also $y \vee z \in I$, i.e. $(y \vee z) \wedge x \leq (y \wedge x) \vee (z \wedge x)$. We have shown that $[a)$ is a distributive lattice thus the nearlattice $\mathcal{N}$ is distributive. □

Example 2 A nearlattice $\mathcal{N}$ depicted in Fig. 3 is not distributive and hence the relative annihilator $\langle a, b\rangle = \{p, q, b, x, y\}$ is not an ideal of $\mathcal{N}$ because $x, b \in \langle a, b\rangle$ but $1 = x \vee b \notin \langle a, b\rangle$.

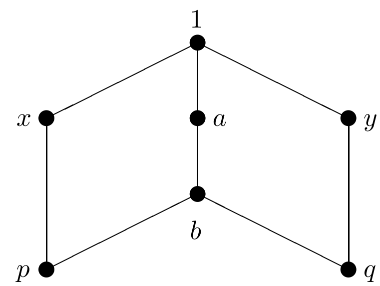

Fig. 3

We say that a nearlattice $\mathcal{N} = (N; \vee)$ is *modular* if each its principal filter is a modular lattice with respect to the induced order $\leq$.

The following result is a generalization of that from [8] for nearlattices:

Theorem 3 *Let $\mathcal{N} = (N; \vee)$ be a nearlattice. The following conditions are equivalent:*

(i) *$\mathcal{N}$ is modular;*

(ii) *$x \vee y \in \langle a, b\rangle$ for each $b \leq a$ and all $x \in (b]$, $y \in \langle a, b\rangle$.*

Proof (i)$\Rightarrow$(ii): Let $y \in \langle a, b\rangle$ for $b \leq a$ and $x \in (b]$, i.e. $x \leq b \leq a$, thus $x, b, a, x \vee y \in [x)$ and, due to modularity of the lattice $[x)$,

$$a \wedge (x \vee y) = (a \wedge y) \vee x \leq b$$

whence $x \vee y \in \langle a, b\rangle$.
(ii)$\Rightarrow$(i): Let $x, y, z \in [a)$ for some $a \in N$ with $x \leq z$. Then $z \wedge y$ exists in $[a)$ and $x \vee (z \wedge y) \leq z$ and $z \wedge x = x \leq x \vee (z \wedge y)$, therefore $x \in \langle z, x \vee (z \wedge y)\rangle$. Further, $z \wedge y \leq x \vee (z \wedge y)$ thus $y \in \langle z, x \vee (z \wedge y)\rangle$. By (ii) we have $x \vee y \in \langle z, x \vee (z \wedge y)\rangle$, i.e. $(x \vee y) \wedge z \leq x \vee (y \wedge z)$ and hence $[a)$ as well as $\mathcal{N}$ is modular. $\square$

Example 3 One can easily see that the nearlattice $\mathcal{N}$ in Fig. 3 is not modular. For $b \leq a$ and for $p \in (b]$, $y \in \langle a, b\rangle$ we have $1 = p \vee y \notin \langle a, b\rangle$.

Let $\mathcal{N} = (N; \vee)$ be a nearlattice and $\emptyset \neq A \subseteq N$. A is called a *sublattice* of $\mathcal{N}$ if it is a lattice with respect to the induced order $\leq$ of $\mathcal{N}$ and $\vee$ and $\wedge$ coincide with the corresponding operations of $\mathcal{N}$.

A sublattice M of a nearlattice $\mathcal{N}$ is called *maximal* if M is not a proper sublattice of another sublattice of $\mathcal{N}$.

From now on, we will suppose that every maximal sublattice M_γ of a nearlattice $\mathcal{N}$ has a least element 0_γ.

We define a nearlattice $(N;\vee)$ to be 0-*distributive* if for all $x,y,z \in M_\gamma$, if $x \wedge y$, $x \wedge z$ are defined and

$$x \wedge y = 0_\gamma = x \wedge z \quad \text{then} \quad x \wedge (y \vee z) = 0_\gamma.$$

Definition 5 Let $\mathcal{N}$ be a nearlattice such that each of its maximal sublattices M_γ has a least element 0_γ. For $a \in N$, define $\langle a \rangle_\gamma = \{y \in M_\gamma; a \wedge y = 0_\gamma\}$, the so-called *annihilator of* a.

Remark 2 It is an easy observation that if a nearlattice $\mathcal{N}$ has a least element 0 (and hence it is a lattice M_1) then we have $\langle a \rangle_1 = \langle a, 0 \rangle$ for each $a \in N$. Moreover, in every nearlattice $\mathcal{N}$ where each maximal sublattice M_γ has a least element 0_γ we have $\langle a \rangle_\gamma = \langle a, 0_\gamma \rangle \cap M_\gamma$ for each $a \in M_\gamma$.

Theorem 4 *Let $\mathcal{N}$ be a nearlattice such that each of its maximal sublattices M_γ has a least element 0_γ. The following conditions are equivalent:*

(i) *every M_γ is 0-distributive;*

(ii) *$\langle a \rangle_\gamma$ is an ideal in M_γ for each $a \in N$ whenever $\langle a \rangle_\gamma \neq \emptyset$.*

Proof (i)⇒(ii): Let $x,y,z \in M_\gamma$ and assume $x \wedge z = 0_\gamma$, $y \wedge z = 0_\gamma$. Due to 0-distributivity of M_γ, also $(x \vee y) \wedge z = 0_\gamma$ and hence $x \vee y \in \langle z \rangle_\gamma$. Of course, if $t \in \langle z \rangle_\gamma$ and $u \leq t$ for $u \in M_\gamma$ then $z \wedge u \leq z \wedge t = 0_\gamma$ whence $u \in \langle z \rangle_\gamma$. Thus $\langle z \rangle_\gamma$ is an ideal of M_γ.

(ii)⇒(i): Let $a,b,c \in M_\gamma$ and $a \wedge c = 0_\gamma$, $b \wedge c = 0_\gamma$. Then $a,b \in \langle c \rangle_\gamma$ and, by (ii), also $a \vee b \in \langle c \rangle_\gamma$, i.e. $(a \vee b) \wedge c = 0_\gamma$ thus M_γ is 0-distributive. □

Example 4 (a) Consider the nearlattice $\mathcal{N} = (N;\vee)$ depicted in Fig. 4.

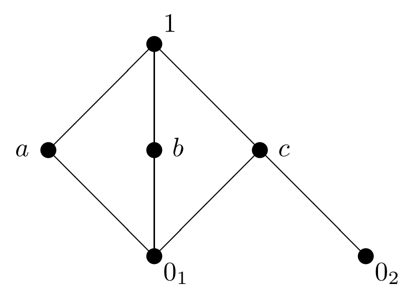

Fig. 4

Clearly, $M_1 = \{0_1, a, b, c, 1\}$ and $M_2 = \{0_2, c, 1\}$ are the only maximal sublattices of $\mathcal{N}$. We have $a \wedge b = b \wedge c = 0_1$, but $b \wedge (a \vee c) = b \wedge 1 = b \neq 0_1$, so M_1 is not 0-distributive, i.e. $\mathcal{N}$ is not 0-distributive. Let us note that for $x \in \{a,b,c\}$, the set $\langle x \rangle_1$ is not an ideal in M_1. On the contrary, M_2 is 0-distributive and for each its element $y \in M_2$, the set $\langle y \rangle_2$ is an ideal in M_2.

(b) It is easy to check that for each $a \in N$ of the nearlattice $\mathcal{N}$ from Example 2 (see Fig. 3), if $\langle a \rangle_\gamma \neq \emptyset$ then it is an ideal in M_γ ($\gamma = 1,2$ and $0_1 = p$, $0_2 = q$). Hence $\mathcal{N}$ is 0-distributive.

References

[1] Abbott, J. C.: *Semi-boolean algebra.* Mat. Vestnik **4** (1967), 177–198.

[2] Beazer, R.: *Hierarchies of distributive lattices satisfying annihilator conditions.* J. London Math. Soc. **11** (1975), 216–222.

[3] Chajda, I., Kolařík, M.: *A decomposition of homomorphic images of nearlattices.* Acta Univ. Palacki. Olomuc., Fac. rer. nat., Math. **45** (2006), 43–52.

[4] Chajda, I., Kolařík, M.: *Nearlattices.* Discrete Math., to appear.

[5] Cornish, W. H.: *The free implicative BCK-extension of a distributive nearlattice.* Math. Japonica **27**, 3 (1982), 279–286.

[6] Cornish, W. H., Noor, A. S. A.: *Standard elements in a nearlattice.* Bull. Austral. Math. Soc. **26**, 2 (1982), 185–213.

[7] Davey, B.: *Some annihilator conditions on distributive lattices.* Algebra Universalis **4** (1974), 316–322.

[8] Davey, B., Nieminen, J.: *Annihilators in modular lattices.* Algebra Universalis **22** (1986), 154–158.

[9] Grätzer, G.: General Lattice Theory. *Birkhäuser Verlag*, Basel, 1978.

[10] Halaš, R.: *Subdirectly irreducible distributive nearlattices.* Math. Notes **7**, 2 (2006), 141–146.

[11] Hickman, R.: *Join algebras.* Communications in Algebra **8** (1980), 1653–1685.

[12] Mandelker, M.: *Relative annihilators in lattices.* Duke Math. J. **40** (1970), 377–386.

[13] Nieminen, J.: *The Jordan–Hölder chain condition and annihilators in finite lattices.* Tsukuba J. Math. **14** (1990), 405–411.

[14] Noor, A. S. A., Cornish, W. H.: *Multipliers on a nearlattices.* Commentationes Mathematicae Universitatis Carolinae (1986), 815–827.

[15] Scholander, M.: *Trees, lattices, order and betweenness.* Proc. Amer. Math. Soc. **3** (1952), 369–381.

[16] Scholander, M.: *Medians and betweenness.* Proc. Amer. Math. Soc. **5** (1954), 801–807.

[17] Scholander, M.: *Medians, lattices and trees.* Proc. Amer. Math. Soc. **5** (1954), 808–812.