Remarks on Existence of Positive Solutions of some Integral Equations

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Abstract

We study the existence of positive solutions of the integral equation

$$x(t) = \mu \int_0^1 k(t,s) f(s, x(s), x'(s), \dots, x^{(n-1)}(s)) \, ds, \quad n \geq 2$$

in both $C^{n-1}[0,1]$ and $W^{n-1,p}[0,1]$ spaces, where $p \geq 1$ and $\mu > 0$. Throughout this paper k is nonnegative but the nonlinearity f may take negative values. The Krasnosielski fixed point theorem on cone is used.

Key words: Positive solutions, Fredholm integral equations, cone, boundary value problems, fixed point theorem.

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4   Introduction

In analyzing nonlinear phenomena many mathematical models give rise to problems for which only nonnegative solutions make sense. This paper deals with existence of positive solutions of the integral equations of the form

$$x(t) = \mu \int_0^1 k(t,s) f(s, x(s), s'(s), \dots, x^{(n-1)}(s)) \, ds, \tag{1.1}$$

where $\mu > 0$ is a constant and $n \geq 2$.