Fixed Point Analysis for Non-oscillatory Solutions of Quasi Linear Ordinary Differential Equations

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Abstract

The paper deals with the quasi-linear ordinary differential equation $(r(t)\varphi(u'))' + g(t,u) = 0$ with $t \in [0,\infty)$. We treat the case when g is not necessarily monotone in its second argument and assume usual conditions on $r(t)$ and $\varphi(u)$. We find necessary and sufficient conditions for the existence of unbounded non-oscillatory solutions. By means of a fixed point technique we investigate their growth, proving the coexistence of solutions with different asymptotic behaviors. The results generalize previous ones due to *Elbert–Kusano*, [Acta Math. Hung. 1990]. In some special cases we are able to show the exact asymptotic growth of these solutions. We apply previous analysis for studying the non-oscillatory problem associated to the equation when $\varphi(u) = u$. Several examples are included.

Key words: Quasi-linear second order equations; unbounded, oscillatory and non-oscillatory solutions; fixed-point techniques.

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