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Join-Closed and Meet-Closed Subsets in Complete Lattices *

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Abstract

To every subset A of a complete lattice L we assign subsets $J(A)$, $M(A)$ and define join-closed and meet-closed sets in L. Some properties of such sets are proved. Join- and meet-closed sets in power-set lattices are characterized. The connections about join-independent (meet-independent) and join-closed (meet-closed) subsets are also presented in this paper.

Key words: Complete lattices, join-closed and meet-closed sets.

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Let $(L,\leq)$ be a complete lattice in which $\bigvee A, \bigwedge A$ denote the supremum and the infimum of any subset $A \subseteq L$, respectively. The least and the greatest elements in $(L,\leq)$ are denoted by 0, 1, respectively. If $A \subseteq L$, $A \neq \emptyset$, then we put $A_x := A \setminus \{x\}$ for $x \in A$ and

$$J(A) = \Big\{\bigvee A_x \mid x \in A\Big\}, \quad M(A) = \Big\{\bigwedge A_x \mid x \in A\Big\}.$$

Instead of $M(J(A))$, $J(M(A))$ we write just $MJ(A), JM(A)$. If we put $P_x = (J(A))_{\bigvee A_x} = \{\bigvee A_a \mid a \in A_x\}$, then $MJ(A) = \{\bigwedge P_x \mid x \in A\}$. Dually, $R_x = (M(A))_{\bigwedge A_x} = \{\bigwedge A_a \mid a \in A_x\}$ and $JM(A) = \{\bigvee R_x \mid x \in A\}$. It is easy to see that $x \leq \bigwedge P_x$ and $\bigvee R_x \leq x$ for all $x \in A$, thus $\bigvee R_x \leq \bigwedge P_x$.

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