

# Density of a Family of Linear Varieties<sup>\*</sup>

GRAZIA RAGUSO, LUIGIA RELLA

*Dipartimento di Matematica, Campus Universitario,  
Via E. Orabona 4, 70125 Bari, Italy  
e-mail: raguso@dm.uniba.it*

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## Abstract

The measurability of the family, made up of the family of plane pairs and the family of lines in 3-dimensional space  $A_3$ , is stated and its density is given.

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## 1 Introduction

A measure on a family of geometric objects can be introduced by assigning to each object a point of an auxiliary space and considering a suitable measure on that space. In general the dimension of the auxiliary space is equal to the number of parameters on which the geometric objects depend. A basic problem is to specify measures which are invariant with respect to a given group of transformations which map the family onto itself.

This problem was first considered by Crofton [3] who specified the invariant measure on the family of all straight lines in Euclidean 2-space  $E^2$ . This was extended to  $E^3$  by Deltheil [4] and Chern [1] first considered families of geometric objects in projective space.

Santaló [9] calculated measures of certain families of varieties with respect to three different groups and found that these were equal. Stoka [10] studied the family of parabolas. He proved that a family is measurable if it is measurable with respect to its maximal group of invariance

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