Acta Univ. Palacki. Olomuc., Fac. rer. nat.,
Mathematica **50**, 2 (2011) 5–12

Thoughts about Selected Models for the Valuation of Real Options

Mikael COLLAN

University of Turku, Turku School of Economics
Pori Unit, PL 17, 28101 Pori, Finland
e-mail: mikael.collan@utu.fi

Dedicated to Lubomír Kubáček on the occasion of his 80th birthday

(Received March 31, 2011)

Abstract

This paper discusses option valuation logic and four selected methods for the valuation of real options in the light of their modeling choices. Two of the selected methods the Datar–Mathews method and the Fuzzy Pay-off Method represent later developments in real option valuation and the Black & Scholes formula and the Binomial model for option pricing the more established methods used in real option valuation. The goal of this paper is to understand the big picture of real option valuation models used today and to discuss modeling perspectives for the future.

Key words: real option valuation, option valuation models

2010 Mathematics Subject Classification: 91G20, 91G60, 91G99

1 Introduction

Real options are the different types possibilities found in connection with real investments that allow managers to capture the potential in the investment, these possibilities are often referred to as managerial flexibility.

As real options are not a thing of fiction, but important real options are often available in real investments it is a matter of interest to managers and business / project owners to be able to understand the value of the real options connected to their investments. Real option value is not only interesting from the point of view of understanding the whole value of an investment, but also, and perhaps especially in situations where comparisons between possible investment alternatives are made. It is also to be noted that the most often used capital budgeting method, the Net Present Value method is not able to consider aspects of investment profitability that are covered by real options analysis [11].

Lately, criticism has been voiced about the application of the 1970's methods for real option valuation and their apparent lack of focus on real world relevance and usability for practitioners [3]. These reasons are sufficient to make real options valuation, models for the valuation of real options, and modeling the valuation of real options an interesting issue for research and relevant for both managers and the academia.

2 Real option valuation as a modeling problem

The option valuation problem, or the logic of option pricing is rather straight forward: the value of an option is the present value of the chance of occurrence weighted expected value of the outcomes of the distribution of the future option values, while mapping the negative values zero. The reason for considering the negative values of the future option value distribution as zero is that the holder of the option has the right, but not the obligation to exercise the option contract. The holder will not exercise if it would cause a loss, but exercises only if profit is created, thus making the downside zero at maximum. The three major components of modeling the value of a real option are:

a) the modeling of the future value distribution
b) the calculation of the expected value of the future value distribution while mapping negative values of the distribution zero, and
c) modeling the calculation of the present value of the expected value.

To be more precise: the modeling of the future value distribution can be interpreted as the modeling of how the future value distribution is created and the calculation of the expected value of the future distribution as the selection of the procedure that is used in the calculation of a single expected value (used as the expected value of the option price).

3 Modeling choices of four selected models used in real option valuation

We have selected four model types and a representative model for each type that are used the valuation of real options for a closer look at the modeling choices that they are based on. The model types and the models selected are:

i) differential equation solutions, represented by the famous, 1997 "Nobel Prize"-winning, Black & Scholes option pricing formula from 1973 that is an often used model for the valuation of real options
ii) discrete event and decision models, represented by the Binomial Option Pricing model by Cox, Ross & Rubinstein from 1979, also a very used model in real option valuation
iii) simulation based methods for option valuation, represented by the Datar–Mathews model for real option valuation from 2004 [7], specifically built

for the valuation of real options to be used in a corporate investment decision-making setting

iv) fuzzy logic based methods, represented the Fuzzy Pay-off Method for real option valuation from 2009 [5], a method built based on using fuzzy numbers to represent the future distribution of expected option value and applying fuzzy mathematics to reach the option value

A short description of the models is given below. Table 1 summarizes the presented models.

Model	Process used to create future value distribution	Distribution type	Discounting of the expected value	Other
Black & Scholes (1973)	Geometric Brownian Motion	Continuous, log-normal probability distribution	Continuous discounting with rf	Closed form solution, replication
Binomial (1979)	Binomial tree process	Quasi log-normal probability distribution	Compound discounting with rf	Backwards iteration to solve value, approaches B & S
Datar–Mathews (2004)	Cash-flow scenarios + MC-simulation	Probability distribution of various shapes	Flexible, user selectable	Practitioner oriented
Fuzzy Pay-off Method (2009)	Cash-flow scenarios ⇒ Creation of fuzzy number	Fuzzy number	Flexible, user selectable	Simplistic, uses fuzzy logic

Tab. 1: Summary of the selected (real)option valuation models

3.1 Black & Scholes option pricing formula 1973

The original Black & Scholes formula [2] is designed to value a European call options contract based on the price of an underlying stock. It is based on a strict set of assumptions regarding the financial markets reality in which it is applicable that allow the treatment of the future development of any traded as a random walk allowing for the use of stochastic processes, in this case the Geometric Brownian Motion (GBM). Using the GBM is a modeling choice that covers also the issue of modeling of the future value distribution of the option, because the GBM effectively defines the resulting distribution.

“The replication argument” is a brilliant observation that is behind the construct of the Black & Scholes formula: any two assets with the same cash-flows and the same risk must be have the same price under perfect markets. Thus, assuming perfect markets with the above assumptions, any combination of securities that are traded in these markets and that delivers exactly the same cash-flow as an option contract must be worth exactly as much as the option contract. Black & Scholes observed that a cash-flow identical to the “option cash-flow” can be reached by a constructing a combination of borrowing money and buying the underlying (stock) in the amount determined by the option delta.

$$\begin{aligned} C &= SN(d_1) - Xe^{-r(T-t)}N(d_2), \\ d_1 &= \frac{\ln\frac{S}{X} + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1 - \sigma\sqrt{T-t}, \end{aligned}$$

where C is the European Call option price, S is the price of the underlying asset, X is the exercise price, $T-t$ is the time to maturity, r is the risk-free rate of return, σ is the volatility, N is the cumulative normal distribution function.

The construct of the Black & Scholes method is very clever, as the choice of assumptions and modeling and the replication argument are such that allow for a closed form solution that returns the call option value as a single number.

The calculation of the expected value of the future value distribution while mapping negative values of the distribution zero, and the calculation of the present value of the expected value are embedded in the closed form solution; the replication argument results in the very elegant way of considering the calculation of the expected value. The discounting back of the “future expected value” is done by using a continuously compounding risk-free rate of return as the rate of discount (in essence for both, the revenue- and the cost-side of the option).

3.2 Binomial option pricing model 1979

The binomial option pricing model [6] is based on the use of a “discrete-time” binomial tree or latice for modeling the price variation of the underlying asset. In other words the creation of the expected value distribution is done by creating a binomial lattice by using a binomial process for stock price changes. The binomial process used allows for two possible directions for the underlying asset value at each time step, up or down, with connected probabilities p and $1-p$. With these assumptions, the price of the European call option C at time $T-n$ follows the the equation [10]

$$C = \frac{1}{(1+r)^n} \sum_{j=a}^{n} \binom{n}{j} p^j (1-p)^{n-j} \left[(1+u)^j (1+d)^{n-j} S_{T-n} - K\right]$$

where C is the price of a European option before expiration (T) until which there are n time steps, S is the underlying asset price (at the indicated time $T-n$; n steps before expiration), K is the strike price of the option, r is the discount rate for each time step, p is the probability of an upward and $1-p$ the probability for a downward movement, a is the summation is the min. number of up-ticks so that the call finishes in-the-money.

Option pricing using the binomial pricing model is a three-stage process: first the binomial tree is constructed, then the option value at each final node (end of maturity) is calculated, and finally the option value for all earlier nodes is calculated by iterating backwards from the final nodes. The assumptions underlying the original binomial pricing model are similar to the assumptions made for the Black & Scholes model and discussed above.

The modeling of the future value distribution is done by using a binomial process for the underlying asset price that results in a discontinuous quasi-log-normal distribution that approaches the continuous distribution that is the result of the GBM process used in the Black & Scholes model. The calculation of the real option value is done by starting from the "end" or final values of the binomial lattice, created by the binomial process as described above. From the final values the earlier node values are calculated, all the way back to the first node that is the real option value at time zero. In the process a compounding risk-free rate of risk is used as the discount rate is used and the rate of compounding is the number of nodes per year. Cost and revenue side of the real option are not separated.

3.3 Datar–Mathews method for real option valuation 2004

The Datar–Mathews method [7, 12, 8] is a simulation based valuation algorithm that has been specifically constructed for the purpose of real option valuation. The method relies on cash-flow scenarios for the operational cash-flows of an investment project that is the real option. The cash-flow scenarios are created by managers and experts in charge of the project. The cash-flow scenarios are used an input into a Monte Carlo simulation that is used to create a probability disttribution of the expected net present value for the project under analysis, i.e. the real option. This distribution is also known as the pay-off distribution. The present value distribution is calculated by using (allowing the use of) separate discount rates for the revenues and the costs. Real option value is calculated from the pay-off distribution by finding the probability weighted mean while mapping the negative pay-off distribution values zero. The intuition of the Datar-Mathews method in a nut-shell can be expressed as [8]:

$$\text{Real Option Value} = \text{Risk Adjusted Success Probability} * (\text{Benefits} - \text{Costs})$$

The the Datar–Mathews method does not rely on a selection and the use of a predetermined process for the modeling (forecasting) of the future distribution for the underlying asset value. The modeling of the future value distribution is done by using (normative) managerial cash-flow information, given in the form of cash-flow scenarios, as the basis for a Monte Carlo simulation that is used to create a probability distribution of the expected net present value of the project in question, i.e. the real option.

As the method "exists" as a spread-sheet application the discounting of the future value distribution can be automated and the options available are very flexile. Due to the flexible construct of the method also non-lognormal cash-flow distributions can be quite easily accomodated, this is better in line with the reality of real options [8]. The calculation of the real option value is done by calculating the probability weighted mean of the pay-off distribution while mapping negative values of the pay-off distribution zero. The Datar–Mathews method result converges with the Black & Scholes result when the Monte Carlo is run enough times.

3.4 Fuzzy pay-off method for real option valuation 2009

Fuzzy pay-off method (FPOM) [5, 4] is the latest addition of the four to the real option valuation method arsenal. The method is based on a similar construct as the Datar–Mathews method, it relies on cash-flow scenarios as a basis for creation of a net present value distribution for the real option under analysis. The difference with the other presented methods is that the method uses the cash-flow scenarios in the creation of a pay-off distribution that is treated as a fuzzy number and does not treat the distribution as a probability distribution. The method is applicable to any shape of pay-off distributions, but simple triangular or trapezoidal distributions are most straight-forward to use. Net present value is calculated for each one of these scenarios. The discount rates can be selected separately for costs and for revenues and the selection of the rate of compounding is left to the analyst. A pay-off distribution is then created from the cash-flow scenario NPVs. For more information on how the pay-off distribution is created see, e.g. [5].

This procedure results in a triangular fuzzy number that is the pay-off distribution for the real option under analysis and that is used as real option the future value distribution. The fuzzy pay-off method calculates the real option value from the pay-off distribution (fuzzy NPV) as follows:

$$ROV = \frac{\int_0^\infty A(x)\,dx}{\int_{-\infty}^\infty A(x)\,dx} \times E(A_+)$$

where A stands for the pay-off distribution (fuzzy NPV), $E(A_+)$ denotes the possibilistic mean value of the positive side of the pay-off distribution and $\int_{-\infty}^\infty A(x)\,dx$ computes the area below the whole pay-off distribution, and $\int_0^\infty A(x)\,dx$ computes the area below the positive part of the pay-off distribution.

The structure of the pay-off method is in line with the option valuation logic of the classical option valuation methods, and especially with the Datar–Mathews method. Tab. 1 summarizes the presented models.

4 Conclusions and some ideas for future modeling of real option valuation

The four types of models for the valuation of real options all offer different insights into the variety of methods that can be used in framing the same problem. Three of the four models use a probability distribution in expressing the future distribution of the real option value one uses a fuzzy number. In broader terms the use of the probability distribution is greatly more widespread, perhaps due to the more widespread knowledge and experience on using probability distributions.

The Datar–Mathews model and the Fuzzy pay-off model use expert generated cash-flow scenarios as an input into real option valuation. This allows for the process of future value distribution creation to include information that is outside of the "flexibility" offered by a pre-determined process, for example information about hedging strategies used for taking out risks. This use of managerial knowledge brings real option valuation closer to real life. The requirements for the models seems to have been set on a new level: real option valuation models should be able to cope with the requirements of the real world that is to have tolerance for the many imperfections of real asset reality in comparison to the theoretical complete and efficient markets. This gives a possibility and motivation to think about some new ways of modeling the real option valuation problem or at least to look at some of the components used in real option valuation with an open mind. From a mathematical modeling perspective it is interesting to think about new approaches for framing the option valuation problem. Indeed there are many possible feasible alternatives for example, for the creation of the future option value distribution. As the number of possible new avenues is great the presentation here is limited to some shortly presented ideas applicable to real option valuation models in the future.

The source of good information for real option valuation is more often than not experts, as existence of any historical data sets, or collected data for similar previous endeavours is not available, it is the human knowledge about our real options that we must seek. Taking into consideration the limitations of the available information is a task that real option modeling should look at, apart from using stochastic models and fuzzy logic that have both been already tested we could also look at *subjective probability* [1, 13] and *credibility* [9] as a frame for our thought when we look at the real option valuation problem. These are "other" ways to define uncertainty—other than probability and fuzzy logic and we should try them out!

Using simulation in creating a distribution for the future value of a real option is an established methodology, but has anyone "used a manager to draw" the distribution as it is seen to reflect reality and, then running a *curve fitting algorithm* that would treat the distribution as a curve and find an "as close as possible" fitting defined curve or a function to fully define the distribution? To the best of my knowledge such an attempt has not been reported.

Real option valuators should put more emphasis on the intuitive and understandable *presentation of results* and perhaps show more than "just the real option value". The whole process of real option valuation itself contains information about the real option and is likely to often be of interest to the decision-makers—yet they are most often shut out of that information. Presenting real option valuation results as if they were coming from a black box is not only an inferior way to use the obtained results, but is also prone to cause the rejection of the method by managers, who often want to understand the tools used in the evaluation. This IS also the modelers' problem even if the modeler may not be the analyst using and presenting the model.

It is with this challenge for scholars and managers interested in mathematical modeling of real options that I close this paper: "I dare you to create new models for option valuation that challenge old assumptions and perhaps offer a better fit to the reality of real options".

References

[1] Anscombe, F. J., Aumann, R. J.: *A definition of subjective probability.* The Annals of Mathematical Statistics **34** (1963)), 199–205.

[2] Black, F., Scholes, M.: *The pricing of options and corporate liabilities.* Journal of Political Economy **81** (1973), 637–659.

[3] Borison, A.: *Real options analysis: Where are the emperor's clothes?.* Journal of Applied Corporate Finance **17** (2005), 17–31.

[4] Collan, M., Fullér, R., Mezei, J.: A Fuzzy Pay-Off Method for Real Option Valuation. In: IEEE International Conference on Business Intelligence and Financial Engineering, *IEEE*, Beijing, PRC, 2009, 165–169.

[5] Collan, M., Fullér, R., Mezei, J.: *Fuzzy pay-off method for real option valuation.* Journal of Applied Mathematics and Decision Systems **2009**, doi: 10.1155/2009/238196 (2009), Article ID 238196, 1–14.

[6] Cox, J., Ross, S., Rubinstein, M.: *Option pricing: A simplified approach.* Journal of Financial Economics **7** (1979), 229–263.

[7] Datar, V., Mathews, S.: *European real options: An intuitive algorithm for the black Scholes formula.* Journal of Applied Finance **14** (2004), 7–13.

[8] Datar, V., Mathews, S., Johnson, B.: *A practical method for valuing real options: The Boeing approach.* Journal of Applied Corporate Finance **19** (2007), 95–104.

[9] Dorweiler, P.: *A survey on risk credibility in experience rating.* Proceedings of the Casualty Actuarial Society **21** (1934), 1–25.

[10] Dubofsky, D. A., Miller, T. W.: Derivatives – Valuation and Risk Management. *Oxford University Press*, New York, 2003.

[11] Kulatilaka, N., Marcus, A.: *Project valuation under uncertainty: When does DCF fail?.* Journal of Applied Corporate Finance **5** (1992), 92–100.

[12] Nau, R.: *De Finetti was right: probability does not exist.* Theory and Decision **51** (2002), 89–124.